

## **Inflationary universe in Kaluza-Klein theories**

A. S. Majumdar

S.N.Bose National Centre for Basic Sciences

Block JD, Sector III, Salt Lake, Calcutta 700091, India

**Abstract.** We describe extended inflation and its typical problems. We then briefly review essential features of Kaluza-Klein theory, and show that it leads to a scenario of inflationary cosmology in four dimensions. The problem of stable compactification of extra spatial dimensions is discussed. The requirements for successful extended inflation lead to constraints on the parameters of higher dimensional models.

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### **1. Introduction**

The standard big-bang model [1] describing a homogeneous and isotropic universe with Friedmann-Robertson-Walker (FRW) metric, is to date, the most successful model of cosmology. However, it is well known that the standard cosmological model is plagued with certain key problems like the horizon, flatness, entropy, and structure formation problems [2]. The most plausible solution to these and other problems of this model is provided by the inflationary scenario [3]. Inflation entails a period of expansion of the cosmic scale factor by at least a factor of  $10^{28}$ . The widely accepted mechanism to drive this large expansion is through the means of a scalar field stuck in a local minimum of its potential with a huge energy density. The 'old' inflationary model [4] is based on a first order phase transition of a scalar field, as it tunnels through the potential barrier towards the absolute minimum. The transition proceeds via nucleation of bubbles of the true vacuum phase. During this process the dominant vacuum energy of the scalar field drives an exponential expansion of the scale factor. The bubbles of the new phase are unable to meet in an expanding background and hence, the true vacuum never percolates. Thus one faces the "graceful exit problem" [5].

A variant of this scenario, i.e., the so called 'new' inflationary model [6], relies on a slow rollover phase transition of the scalar field to solve the graceful

exit problem. However, here one requires the effective potential to be extremely flat near the origin. This necessitates large fine-tuning of the potential parameters. Similar fine-tuning problems are also encountered in 'chaotic' inflation [7].

Extended inflation (EI) [8-9] restores the spirit of 'old' inflation in the sense that the scalar field herein undergoes a strongly first order phase transition. However, here the Einstein theory of gravity is replaced in general by scalar-tensor theories [10-12]. The simplest such theory is the Jordan-Brans-Dicke (JBD) theory [13-14], which was used by La and Steinhardt [8-9] for obtaining a model of EI which could remove the graceful exit problem of 'old' inflation. However, it was soon realized that this simple model of EI had some typical problems of its own, and the search continued for better models.

In this review we shall first set up the framework used for EI in context of the simple JBD model. In doing so, we shall see how certain basic problems crop up in implementing EI (Section 2). As we shall note, a viable EI scenario is naturally incorporated for the 4-dimensional spacetime of higher dimensional (Kaluza-Klein) (KK) theories. In Section 3 we shall briefly introduce the rudimentary features of KK theories before showing how inflation is obtained for the 4-dimensional spacetime of one simple KK model. An essential feature of KK theories is that the extra spatial dimensions are compactified to a tiny radius. We shall see how this occurs for the case of more sophisticated models in Section 4. The requirement of obtaining viable density perturbations and observations on the cosmic microwave background radiation (CMBR) places severe restrictions on the potential of the scalar field used in inflation. We conclude by describing the constraints imposed on the parameters of KK models by the conditions for successful EI in Section 5.

## 2. Extended inflation and its problems

Extended inflation was first implemented [8-9] using a simple JBD model coupled to matter fields, having an action given by

$$S = \int d^4x (-g)^{1/2} [-\phi R + \omega g^{\mu\nu} \frac{\partial_\mu \phi \partial_\nu \phi}{\phi} + 16\pi \mathcal{L}_{matter}] \quad (1)$$

The JBD field  $\phi$  plays the role of a time varying gravitational constant.  $\omega$  is a free parameter called the JBD parameter. The Einstein theory follows in the limit  $\omega \rightarrow \infty$  [10-12]. The present day accuracy of time delay experiments require  $\omega > 500$  [15]. The matter sector  $\mathcal{L}_{matter}$  is assumed to contain a scalar field  $\chi$  which undergoes a first order phase transition from a metastable state of its potential (false vacuum) to the energetically favorable absolute minimum (true vacuum). The spacetime geometry is described by a Friedmann-Robertson-Walker (FRW) metric

$$ds^2 = dt^2 - a^2(t) \left[ \frac{dr^2}{1 - kr^2} + r^2 d\Omega^2 \right] \quad (2)$$

where  $a(t)$  is the FRW scale factor and  $k = -1, 0, +1$  denotes an open, flat and closed geometry respectively. The equations of motion that follow from (1) are

$$\begin{aligned}\frac{\dot{a}^2}{a^2} &= \frac{8\pi\rho}{3\phi} - \frac{k}{a^2} + \frac{\omega\dot{\phi}^2}{6\phi^2} - \frac{\dot{a}\phi}{a\phi} \\ \ddot{\phi} + \frac{3\dot{a}\dot{\phi}}{a} &= \frac{8\pi(\rho - 3p)}{3 + 2\omega}\end{aligned}\quad (3)$$

where  $\rho$  and  $p$  are the energy density and pressure of matter respectively. During the inflationary era the energy density is dominated by the vacuum energy of the inflaton field and the equation of state is given by  $p = -\rho$ . The term  $k/a$  is insignificantly small and the following solutions are obtained.

$$\phi(t) = m_{pl}^2 \left(1 + \frac{c_1 t}{c_2}\right)^2; \quad a(t) = \left(1 + \frac{c_1 t}{c_2}\right)^{\omega+1/2} \quad (4)$$

with  $c_1^2 = (8\pi\rho)/3m_{pl}^2$  and  $c_2^2 = (3 + 2\omega)(5 + 6\omega)/12$ . Here  $m_{pl}^2$  is a constant of integration corresponding to the effective Planck mass at the beginning of inflation ( $t = 0$ ). The actual Planck mass should correspond to the current value of  $\phi$  [41]. ( $G^{-1} = m_{pl}^2$ , in units of  $c = \hbar = 1$ ).

In the limit of  $\omega \rightarrow \infty$  equations (4) give a constant solution  $\phi = m_{pl}^2$ , and an exponential solution for the scale factor  $a(t)$ , thus reproducing the 'old' inflationary scenario [3]. However, for small enough  $\omega$  defined by the condition,  $c_1 t > c_2$ , the solutions take the shape

$$\phi(t) = m_{pl}^2 \left(\frac{c_1 t}{c_2}\right)^2 \quad a(t) = \left(\frac{c_1 t}{c_2}\right)^{\omega+1/2} \quad (5)$$

This power law expansion of the scale factor continues till the completion of the phase transition of the inflaton field. Bubbles of true vacuum are produced and their nucleation rate per unit volume can be expressed as  $\Gamma = A \exp(-B)$  where  $A$  is a constant with units of (mass) and  $B$  is the action corresponding to the least resistance path across the potential barrier. This action is called the bounce action [16-17]. Taking into account the expansion of the universe, the stage of progress of the first order phase transition is determined by the parameter  $\epsilon(t)$  given by

$$\epsilon(t) = \overline{H^4(t)} \quad (6)$$

where  $H(t) \equiv \dot{a}(t)/a(t)$ .  $\epsilon$  reaching order unity denotes completion of the phase transition. Nevertheless,  $\epsilon$  should be small enough initially to allow for more than 66 e-foldings ( $e^{66} \sim 10^{28}$ ) of the scale factor necessary to solve the horizon problem. The nucleation rate  $\Gamma$  depends upon the parameters of the

inflaton potential and is a constant here. For a power law expansion,  $H$  goes as  $1/t$  and hence  $\epsilon(t)$  keeps on increasing continuously to reach order unity, at which point the universe is dominated by bubbles of true vacuum. These bubbles collide with each other, with the energy in their walls being converted into the thermal energy of particles and radiation. The universe is reheated and henceforth its evolution proceeds as in a standard radiation dominated era. The JBD field  $\phi$  keeps on growing slowly to reach its current value of  $m_{pl}^2$ .

Although the percolation of true vacuum is possible, this scenario leads to a nearly flat bubble distribution [18-19]. This means that at any given instant, bubbles with all sorts of sizes are present. This can hamper efficient thermalization. Also, the initially produced bubbles can grow out to large sizes, which can later create unwanted inhomogeneities in the CMBR. The fraction of volume occupied by bubbles greater than a certain radius  $r$ , at the end of the phase transition ( $t_{end}$ ) is given by [18-19]

$$f(x > r, t_{end}) \approx (H.x)^{-4/\omega} \quad (7)$$

For  $H.x \sim 10^{28}$ ,  $f$  should be less than  $10^{-4}$  to satisfy the CMBR constraints. This immediately leads to a bound on  $\omega$ , i.e.,  $\omega < 20$ . But as mentioned earlier, JBD theory is consistent with observations only if  $\omega > 500$ . This contradiction is called the  $\omega$  problem.

Several ways have since been suggested to save extended inflation. Among these are variable  $\omega$  theories [20-22], wherein  $\omega$  is a function of the JBD field as incorporated in more general scalar-tensor theories [10-12]. However, the motivation needed to consider such theories is not clear. Modification of the inflaton sector has been considered by certain authors [20-23], with the aim of making the nucleation rate  $\Gamma$  time dependent, thereby lifting the upper bound on  $\omega$  to an acceptable value. Several efforts have gone in the direction of building JBD models with ad hoc potentials for  $\phi$  [24-26]. The philosophy behind such constructions is to allow  $\phi$  to be anchored by these potentials after completion of the inflationary phase transition. In this way an effective Einstein theory would emerge, eliminating the need for the lower bound on  $\omega$ , i.e.,  $\omega > 500$ .  $\omega$  can be as small as required to facilitate the completion of phase transition together with a bubble distribution allowed by the CMBR constraints [18-19].

The sort of scheme mentioned last seems promising enough if implemented in a more natural framework, i.e., without having to include ad hoc potentials. In the next sections we shall see that dimensional reduction of several Kaluza-Klein models give rise to JBD type actions in four dimensions with potential terms for the JBD field  $\phi$ .

### 3. Kaluza-Klein theory and inflation in four dimensions

The study of physics in more than four dimensions received a great boost with the phenomenal success of string theory. Implementation of the concept

of unification has opened up the exciting possibility that our world might consist of extra spatial dimensions. The implications of this for cosmology are profound. For the present context, we would like to see how a scenario of inflationary cosmology for the 4-dimensional spacetime which can get rid of the problems of extended inflation, mentioned earlier, can be obtained. Further, it needs to be explained how there should be an enormous difference in size between the observed four-dimensional spacetime, and the unobservable tightly curled up extra dimensions.

With this aim in mind, we proceed to study inflationary cosmology in higher dimensional theories as seen from the four dimensional point of view. To do this, one needs to establish the correspondence of a Kaluza-Klein theory [27-29] to the language of the standard four dimensional field theory. More precisely, the meaning of the various field operators in a dimensionally reduced field theory has to be brought out clearly [30]. To illustrate this point, let us consider the simple example of a massless scalar field in  $4+D$  dimensions [31]. The corresponding wave equation can be written down as

$$\square_{4+D}\phi \equiv \square_4\phi + \nabla_D\phi = 0 \quad (8)$$

where  $\square_4$  denotes the d'Alembertian in four dimensions and  $\nabla_D$  the Laplacian in  $D$  dimensions (the additional dimensions are all assumed to be spacelike). Assuming the  $D$ -dimensional manifold to be a closed one, we can now expand  $\phi$  in terms of the eigenfunctions of the Laplacian  $\nabla_D$ :

$$\phi(x, y) = \sum_{\nu} u_{\nu}(y) \phi_{\nu}(x) \quad (9)$$

where  $x$  and  $y$  represent the 4-dimensional and  $D$ -dimensional coordinates respectively.  $u_{\nu}$  is the  $\nu$ -th eigenfunction of  $\nabla_D$  with eigenvalue  $\lambda_{\nu}$

$$\nabla_D u_{\nu} = \lambda_{\nu} u_{\nu} \quad (10)$$

The fields  $\phi_{\nu}(x)$  depend only upon the coordinates of the four dimensional spacetime and obey the wave equation given by (from (8),(9),(10))

$$(\square_4 + \lambda_{\nu})\phi_{\nu} = 0 \quad (11)$$

Thus,  $\lambda_{\nu}$  will be observed as masses of the 4-dimensional fields  $\phi_{\nu}$ . These masses are typically of the order  $1/b$  [30-31], where  $b$  represents the size of the compact manifold. Every single massless field in  $4+D$  dimensions therefore appears as a tower of massive states plus some massless modes given by  $\lambda_{\nu} = 0$ . If  $b$  is of the order of planck length, the massive states will not play any role in the dynamics and can be integrated out of the low energy effective field theory. Similar results also hold true for particles of any spin and mass [27-31]. This procedure of dimensional reduction is thus applied by the harmonic

expansion of higher dimensional fields on the compact manifold, to obtain the 4-dimensional effective field theory, retaining only the zero modes.

We can now proceed to analyze our first KK model. This model was considered in detail by Holman et al [32] and shall be used by us to set up the problem of extended inflation in a proper perspective. The action in  $4 + D$  dimensions is given by

$$\tilde{S} = \int d^{4+D}z (-\tilde{g})^{(1/2)} \left[ -\frac{1}{16\pi\tilde{G}} \tilde{R} + \frac{1}{2} \tilde{g}^{MN} \partial_M \tilde{\chi} \partial_N \tilde{\chi} - \tilde{U}(\tilde{\chi}) \right] \quad (12)$$

We have used  $z$  to represent the coordinates of the full  $4 + D$  dimensional spacetime and tildes to mark the various  $4 + D$  dimensional objects. Upper case Latin indices denote the full  $4 + D$  dimensional coordinates, whereas, Greek indices denote 4-dimensional coordinates. The scalar field  $\tilde{\chi}$  plays the role of the inflaton, and it will be assumed to be caught in a metastable state of its potential  $\tilde{U}(\tilde{\chi})$ , from where it can tunnel out by the nucleation of bubbles of the true vacuum. The  $4 + D$  dimensional line element is assumed to take the form

$$d\tilde{s}^2 = dt^2 - a^2(t) d\Omega_3^2 - b^2(t) d\Omega_D^2 \quad (13)$$

with  $d\Omega_3^2$  corresponding to the line element of a maximally symmetric 3-space with scale factor  $a(t)$  and  $d\Omega_D^2$  to that of a D-sphere with scale factor  $b(t)$ . As discussed earlier, the zeroth mode in the harmonic expansion of  $\tilde{\chi}$  is independent of the coordinates of the D-sphere. This enables one to rewrite  $\tilde{S}$  as

$$\tilde{S} = \left[ \int d^D y (g_D(y))^{1/2} \right] S \quad (14)$$

where  $g_D(y)$  is the determinant of the D-dimensional metric.  $S$  is the effective four dimensional action which is independent of the D-coordinates represented by  $y$ .  $S$  is given by

$$S = \int d^4x (-g)^{1/2} \Omega_D b^D(t) \left[ -\frac{R}{16\pi\tilde{G}} - \frac{D(D-1)}{16\pi\tilde{G}b^2} (g^{\mu\nu} \partial_\mu b \partial_\nu b - 1) + \frac{1}{2} g^{\mu\nu} \partial_\mu \chi \partial_\nu \chi - U(\chi) \right] \quad (15)$$

$R$  is the curvature scalar for the four dimensional spacetime with metric  $g_{\mu\nu}(x)$  and  $\Omega_D = (2\pi^{(D+1)/2})/\Gamma(D+1)/2$ . Note here that the kinetic term for the  $b$ -field has appeared with a negative sign. It is not apparent that the action of (15) resembles a JBD action. To see that, it is necessary to make the following redefinitions:

$$\begin{aligned} \frac{\Omega_D b_0^D}{16\pi\tilde{G}} &\equiv \frac{1}{16\pi G} ; \quad \sigma \equiv (\Omega_D b_0^D)^{1/2} \chi \\ V(\sigma) &\equiv (\Omega_D b_0^D) U(\chi) ; \quad \phi \equiv \frac{(b/b_0)^D}{16\pi G} \end{aligned} \quad (16)$$

$b_0$  is a constant which sets the scale of compactification.  $G$  is the four dimensional Newton constant. The field  $\sigma$  shall henceforth be called the inflaton field.  $\phi$  plays the role of a JBD field when we rewrite  $S$  as

$$S = \int d^4x (-g)^{1/2} [-\phi R - \omega g^{\mu\nu} \frac{\partial_\mu \phi \partial_\nu \phi}{\phi} + \alpha \phi^{1-2/D} + 16\pi G \phi (\frac{1}{2} g^{\mu\nu} \partial_\mu \sigma \partial_\nu \sigma - V(\sigma))] \quad (17)$$

where,

$$\alpha = D(D-1)(16\pi G)^{-2/D} b_0^{-2} ; \quad \omega = 1 - 1/D \quad (18)$$

It is important to note several facts about the JBD type action of equation (17). First, as mentioned earlier, the kinetic energy term for the field  $\phi$  occurs with a negative sign in contrast to the usual JBD theories. Secondly, a potential for  $\phi$  (third term) has naturally followed. Thirdly,  $\phi - \sigma$  cross terms are also present. This is an outcrop of dimensional reduction from higher dimensions and as such has no analogue in JBD lagrangians written down directly in four dimensions.

At this stage it is essential to look into the dynamics of this system to check whether extended inflation works here. As usual, it is assumed that the inflaton field  $\sigma$  makes a first order phase transition from its metastable vacuum at  $\sigma = 0$  to the true vacuum at  $\sigma = \sigma_0$ . Defining the value of the potential at the false vacuum as  $V_0 = 8\pi G V(\sigma = 0)$ , we have the following equations of motion from (17):

$$\frac{\dot{a}^2}{a^2} + \frac{\dot{a}\dot{\phi}}{a\phi} + \frac{\omega\dot{\phi}^2}{6\phi^2} + \frac{\alpha\phi^{1-2/D}}{6} - \frac{V_0}{3} = 0$$

$$\ddot{\phi} + \frac{3\dot{a}\dot{\phi}}{a} = -\frac{dW(\phi)}{d\phi} \quad (19)$$

with the potential  $W(\phi)$  given by

$$W(\phi) = \frac{\alpha\phi^{2(1-1/D)}}{2(1-1/D)} - \frac{V_0\phi^2}{1+2/D} \quad (20)$$

Before attempting to solve these equations for  $a(t)$  and  $\phi(t)$ , it is better to briefly recall our purpose here. We would like to obtain expanding or inflationary solutions for the 4-dimensional scale factor  $a(t)$ . Also since  $\phi(t)$  is proportional to the D-dimensional scale factor  $b(t)$ , we would want decreasing solutions for  $\phi(t)$ . This would signify contraction of the extra space. For successful percolation of the true phase, we desire the inflaton phase transition to take place in a suitable manner as stated in section 1.

The potential  $W(\phi)$  (20) is dominated by the second term for large values of  $\phi$ . It has a maximum at  $\phi_0 = ((D+2)/2D)^{D/2}$ . If the initial value of  $\phi$  is greater than  $\phi_0$ , then the field  $\phi$  will roll down the potential hill towards larger values of  $\phi$ , signifying expansion of the  $D$ -dimensional space. If  $\phi$  starts out with a value lesser than  $\phi_0$ , a simple numerical integration of equations (19) shows that enough expansion of the scale factor  $a(t)$  is not possible, before  $\phi$  is driven down to zero [32]. Hence, simultaneous inflation of the 4-dimensional spacetime and compactification of the extra space is not possible in this model.

Notwithstanding the fact that this model does not work, it is interesting to note its one successful feature, namely the nature of phase transition of the inflaton field  $\sigma$ . It is well appreciated that the calculation of the nucleation rate of true vacuum bubbles is quite complicated in models containing more than one time-varying field. The present case is one such example since here the phase transition of  $\sigma$  takes place in the background of an evolving field  $\phi$ . Due to the  $\phi - \sigma$  cross terms in (17), the nucleation rate becomes time dependent. The non-flat nature of the background spacetime complicates the matter further. However, it was observed in refs. [33-34], that in the limit of weak gravity it is possible to systematically "freeze-out" the gravitational effects in the bounce action. For theories derivable from an action of the type given in (17), they obtained a closed form expression for the nucleation rate

$$\Gamma = A_0(16\pi G\phi)^2 \exp(-16\pi G\phi B_0) \quad (21)$$

with  $B_0$  the bounce action as calculated in flat space and  $A$  is proportional to  $\sigma_0^4$ . It should be noted that the approximation of weak gravity is valid only for large  $\phi$ .

The major difference of this theory from 4-dimensional EI models is brought out by noting that both the numerator and denominator in the expression  $\epsilon = \Gamma/H^4$  are now time-dependent. Any solution of this theory which proceeds towards compactification of the extra space (solutions for  $\phi$  decreasing with time), would also contribute towards enhancing the rate of true vacuum percolation. (It can be seen that  $\phi$  is present in the exponent of the expression for  $G$  (21)). This would make it easier for  $\epsilon(t)$  to start out from very small values and still reach order unity to establish completion of the phase transition.

#### 4. Stable compactification and successful inflation

We have seen in the previous section that a simple model of Einstein gravity together with an inflaton field in higher dimensions, does not produce all the features necessary for a viable model of EI in four dimensions. The difficulty can be summed up as a lack of compatibility of obtaining sufficient inflation along with compactification of the extra manifold. A natural extension is to consider a higher dimensional model with a nonminimally coupled inflaton



field. The relevance of nonminimal coupling for scalar fields in Kaluza-Klein theories has been observed by Sunahara et al [35]. They found that certain range of values of the coupling parameter could automatically prohibit the isotropic expansion of all the 4+D dimensions.

Let us now consider a model in ten dimensions with a nonminimally coupled scalar field having an action given by [36]

$$\hat{S} = \int d^{10}z (-\tilde{g})^{1/2} \left[ -\frac{\tilde{R}}{16\pi\tilde{G}} + \frac{1}{2}\tilde{g}^{MN}\partial_M\tilde{\chi}\partial_N\tilde{\chi} - \xi R(\tilde{\chi}^2 - \tilde{\chi}_0^2) - \tilde{U}(\tilde{\chi}) \right] \quad (22)$$

Here  $\xi$  is an arbitrary nonminimal coupling parameter. One can follow the same procedure of dimensional reduction as described in the previous section. After performing the transformations defined in (16), the effective 4-dimensional action is obtained.

The equations of motion for the 4-dimensional scale factor  $a(t)$ , as well as the 4-dimensional JBD field  $\phi(t)$  (related to the internal scale factor  $b(t)$ ) have been derived in [36]. The potential for the JBD field is given by

$$W(Y) = \frac{3Y^{5/3}}{5} - \frac{3Y^2}{4\delta} \quad (23)$$

where

$$Y = \left(\frac{\alpha}{V_0}\right)^{-1/3}\phi(t) ; \quad \delta = 1 - 16\pi G\xi\sigma_0^2 \quad (24)$$

The equations of motion have been numerically integrated for negative values of  $\delta$ . Rapid expansion of the scale factor  $a(t)$  ensues. Analysis of the potential  $W(Y)$  suggests that the initial value of  $Y$  can be as large as required, to enable completion of greater than 65 e-foldings of the scale factor  $a(t)$ , before  $Y$  is driven down to zero. Numerical integration confirms that one has a scenario of rapid expansion for the 4-dimensional scale factor together with a simultaneous contraction of the extra dimensional scale factor.

A prerequisite for a successful theory of EI, is the suitable completion of the inflationary phase transition. To see that this indeed is the case here, one has to calculate  $\epsilon(t)$  for various stages of the evolution of  $a(t)$  and  $\phi(t)$ . The results show [36] that for a wide choice of initial nucleation rates, it is possible to complete the phase transition before  $\phi$  becomes very small to violate the weak gravity approximation [33-34].

In this sort of a scenario the possibility of finding large-sized leftover bubbles after inflation is extremely rare. The bubble distribution is expected to be highly peaked around a certain critical size, corresponding to the formation of a very large majority of bubbles towards the end of the phase transition when the value of  $\epsilon$  is near 1. Consequently, the thermalization of energy in the bubble walls proceeds in an efficient way. It needs to be emphasized that

that this is a generic feature of these kinds of higher dimensional models, and therefore is insensitive to the choice of various free parameters.

Thus in this model, the inclusion of a nonminimally coupled inflaton field in higher dimensions enables the dynamical contraction of the extra space to occur together with extended inflation in the 4-dimensional spacetime. No upper bound on  $\omega$  is needed in this model to produce a well-behaved bubble distribution. Nevertheless, questions regarding compactification that still remain are: (i) There is nothing in the theory to prevent the shrinking of the extra dimensions to zero. (ii) One has to ensure that decompactifying solutions for the internal scale factor  $b(t) (\sim \phi(t))$  are forbidden, even after the inflaton field has settled down at its true vacuum (i.e.,  $V_0 = 0$ ). We shall now see how these problems are tackled by using a different KK model in ten dimensions.

To this end, a ten dimensional JBD model was considered in [37], which has the action given by

$$\tilde{S} = \frac{1}{16\pi} \int d^{4+D} z (-g)^{1/2} [-\tilde{\Phi} \tilde{R} + \tilde{\omega} \tilde{g}^{MN} \frac{\partial_M \tilde{\Phi} \partial_N \tilde{\Phi}}{\tilde{\Phi}} - \tilde{\Lambda} + \tilde{\mathcal{L}}(\tilde{\psi})] \quad (25)$$

where,  $\tilde{\mathcal{L}}(\tilde{\psi})$  denotes the Lagrangian of the inflaton field.  $\tilde{\Lambda}$  is a cosmological constant and  $\tilde{\omega}$  is the 10-dimensional JBD parameter. As seen in the previous section, additional effects are required to obtain a static solution for the internal space. To ensure that the extra dimensions do not shrink down to zero size, one can include the Casimir energy contribution due to the various fields. Such a term is of the form  $A/b^{4+D}$  [38-39], where  $A$  is a constant.

This model was dimensionally reduced via the same procedure, and the effective 4-dimensional action was studied in the conformally transformed Einstein frame in [37]. As was shown in [40], it is indeed the Pauli metric (in the Einstein frame) which obeys the correct equation of motion for a massless spin-2 graviton in four dimensions, as opposed to the metric in the Jordan frame. In the Einstein frame, one has the further advantage that scalar fields in the matter sector have canonical couplings to the metric, and also standard kinetic terms in four dimensions [37, 40]. Apart from the inflaton, the present model has two more scalar fields (JBD type) in four dimensions, one related to the internal scale factor,  $\phi$ , the other coming from the ten dimensional JBD field  $\Phi$ .

The equations of motion were integrated to obtain exponentially expanding solutions for the four dimensional scale factor  $a(t)$ , together with compactifying solutions for the internal scale factor. Nucleation of true vacuum bubbles proceed in a favorable manner, as in the models studied earlier. At the end of the inflatary phase transition, a radiation dominated era is brought about by demanding the total effective cosmological constant to vanish. With this condition, the potential for  $\phi$  in the post-inflationary era is given by

$$V(\phi) = c_1 e^{-2\Phi+\phi} + c_2 e^{-2\Phi+8\phi/3} - c_3 e^{-\Phi+4\phi/3} \quad (26)$$

where  $c_1, c_2$  and  $c_3$  are constants arising from combinations of parameters of this model [37]. At the end of the inflationary phase transition, the internal scale factor undergoes oscillations about the minimum of its potential with decreasing amplitude. Thus one is subsequently led to a scenario wherein a stable internal manifold with finite size occurs. The Einstein theory of gravitation describes the cosmic evolution of the four dimensional spacetime in the post-inflationary stage.

## 5. Constraints on parameters of Kaluza-Klein models

An unsatisfactory feature of higher dimensional models, as we have seen, is the presence of several 'free' parameters, such as the nonminimal coupling parameter, higher dimensional JBD parameter, strength of the cosmological term, etc. It is natural to expect that considerations for implementing a successful EI scenario would impose restrictions on the values of these parameters. The conditions for successful EI can be summarized [41] as follows.

The primary requirement is that one needs to obtain enough inflation ( $Hx \geq e^{66}$ ), before the completion of the inflationary phase transition for solving the horizon and flatness problems. Secondly, a bubble spectrum should ensue which agrees with the CMBR anisotropy ( $\nabla T/T \sim 10^{-5}$ ) as reported by COBE [42]. The quantum fluctuations of the scalar field trapped in the local minimum of its potential should generate suitable density perturbations that are compatible with the subsequent formation of large scale structure in the universe [43-44]. Apart from these requirements on the physics of the phase transition during the inflationary era, one must recover a radiation dominated universe described by the Einstein theory (or JBD theory with  $\omega > 500$ ) after the end of inflation. One must also obtain the correct value for the four dimensional gravitational constant.

All the above criteria for successful EI have been incorporated in case of KK models described in this paper, and restrictions on their parameters obtained in [45]. As we have seen here, in all these models the 4-dimensional effective action contains a potential for the JBD field with a minimum where it can settle down at the end of inflation. Hence, in essence, an Einstein theory ensues at the end of inflation for the 4-dimensional spacetime, whereas, the internal dimensions are compactified to a small radius. Obtaining the correct value of  $G$  leads to a constraint on a parameter of the higher dimensional action.

However, the stability of compactification cannot be ensured in case of the minimal ten dimensional model, considered in Section 3. A generic feature of such models is a suitable bubble spectrum as demanded by CMBR anisotropy. Hence, this requirement imposes rather weak constraints on the higher dimensional parameters. The most stringent condition is that of appropriate density perturbations. This leads to severe restrictions on the parameters, and it was

observed in [45] that out of the models considered here, only the ten dimensional JBD model is able to meet this requirement within a narrow range of allowed parameter space.

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